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REPORT

Directional coupler design

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Research Department, Engineering Division
THE BRITISH BROADCASTING CORPORATION October 1973

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DIRECTIONAL COUPLER DESIGN

G.W.W. McNally, B.Sc.

Summary

The design and performance of directional couplers are investigated with examples of 0 dB, 6 dB and 15 dB couplers being detailed to illustrate the configurations and construction methods used.

Issued under the authority of

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Head of Research Department

Research Department, Engineering Division,
BRITISH BROADCASTING CORPORATION

(RA-112)

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1. Introduction

Before entering into study of directional couplers it is useful to have a clear understanding of the terminology. Representing a coupler

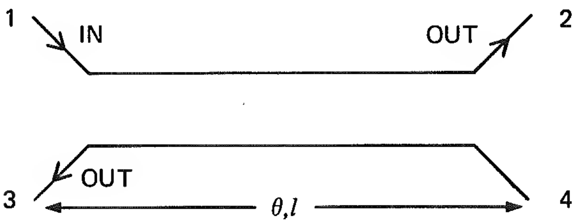


Fig. 1

we define

(a) Output ratio, $K = \dfrac{E_3}{E_2}$

(b) Directivity = $\dfrac{E_3}{E_4}$

(c) Midband coupling, $c = \dfrac{E_3}{E_1}$

(b) and (c) are usually expressed in dB's.

When a wave of voltage amplitude E_1 enters port 1, it is shown in Appendix I that:

$$\frac{E_2}{E_1} = \frac{\sqrt{1-c^2}}{\sqrt{1-c^2}\cos\theta + \mathrm{j}\sin\theta} \tag{1}$$

$$\frac{E_3}{E_1} = \frac{\mathrm{j}\,c\sin\theta}{\sqrt{1-c^2}\cos\theta + \mathrm{j}\sin\theta} \tag{2}$$

$$\frac{E_4}{E_1} = 0 \tag{3}$$

These equations illustrate the properties of the basic directional coupler, namely:—

(a) The coupling and hence output ratio vary sinusoidally with frequency for small values of c and are maximum for $\theta = {}^{\pi}/2$ or $l = {}^{\lambda}/4$.

(b) The directivity is theoretically infinite

(c) The output at port 3 leads the output of port 2 by 90°.

There are many configurations for coupled TEM lines but only two will be considered here.

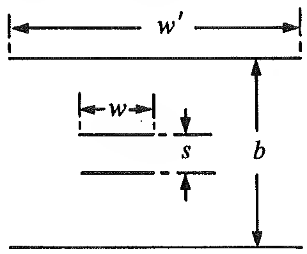


Fig. 2(a)

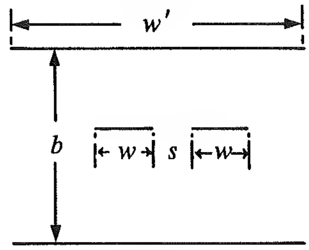


Fig. 2(b)

Fig. 2(a) shows broadside coupled strips which can be used in tight coupling applications (0 dB ⟶ 10 dB) and Fig. 2(b) the coplanar configuration for loose coupling (less than 8 dB). Both cross-sections lend themselves to photo-etched stripline techniques and the calculations assume infinitely thin conductors. These calculations are greatly simplified by the concept of 'odd' and 'even' modes of propagation in coupled transmission lines. In the EVEN mode of excitation current and voltages in the two conductors are equal and of the same sign, while in the ODD mode, the respective voltage and currents are equal but of opposite sign. Referring to Fig. 12, and observing the neutral planes, we get two characteristic impedances:

Z_{oe} – the characteristic impedance of one of the coupling strips to ground in the even mode.

Z_{oo} – characteristic impedance of one of the coupling strips to ground in the odd mode.

All relevant data can be calculated in terms of these impedances. In particular if the coupler is to be matched at all frequencies the input impedance, Zin, must be equal to Z_o. It is shown in Appendix I that to satisfy this condition

$$Z_o = \sqrt{Z_{oo} Z_{oe}} \tag{4}$$

It is also shown that

$$Z_{oe} = Z_o \sqrt{\frac{1+c}{1-c}} \quad \text{and} \quad Z_{oo} = Z_o \sqrt{\frac{1-c}{1+c}} \qquad (5)\ (6)$$

The following sections indicate how dimensions can be obtained to give the required odd/even impedances and hence the required coupling in a 50Ω system.

2. Broadside coupled strips

2.1. Formulae

It has been shown by Cohn[1] that the odd and even mode impedances for this system of conductors are

$$Z_{oe} = \frac{60\pi}{\sqrt{\epsilon_r}} \frac{K(k'_e)}{K(k_e)} \tag{7}$$

$$Z_{oo} = \frac{296 \cdot 1}{\sqrt{\epsilon_r}\, \frac{b}{s} \tanh^{-1} k} \tag{8}$$

where k_e = a parameter

$$k'_e = \sqrt{1 - k^2_e}$$

$K(k_e)$ $K(k'_e)$ = complete elliptic integrals of the first kind.[2] (See Appendix II). If the conditions $(w/b)/1 - s/b) \geqslant 0{\cdot}35$ and $w/s \geqslant 0{\cdot}35$ are satisfied, the fringing capacitances at opposite edges of the strips are independent of strip width and the characteristic impedances are given by:–

$$Z_{oe} = \frac{60\pi/\sqrt{\epsilon_r}}{\frac{w/b}{1-s/b} + \frac{C'_{fe}}{\epsilon_r}} \tag{9}$$

$$Z_{oo} = \frac{60\pi/\sqrt{\epsilon_r}}{\frac{w/b}{1-s/b} + \frac{w}{s} + \frac{C'_{fo}}{\epsilon_r}} \tag{10}$$

where C'_{fe}, C'_{fo} are the fringing capacitances per unit length in the even and odd modes respectively. Fig. 3 gives the values of these capacitances as a function of s/b.

2.2. Design of 6 dB coupler

The configuration of Fig. 2(*a*) for tight coupling is used. The electrical length (l) of the coupling region is chosen to be a quarter wavelength at midband i.e.

$$l = \frac{3.10^8}{4 f_c \sqrt{\epsilon_r}} \tag{11}$$

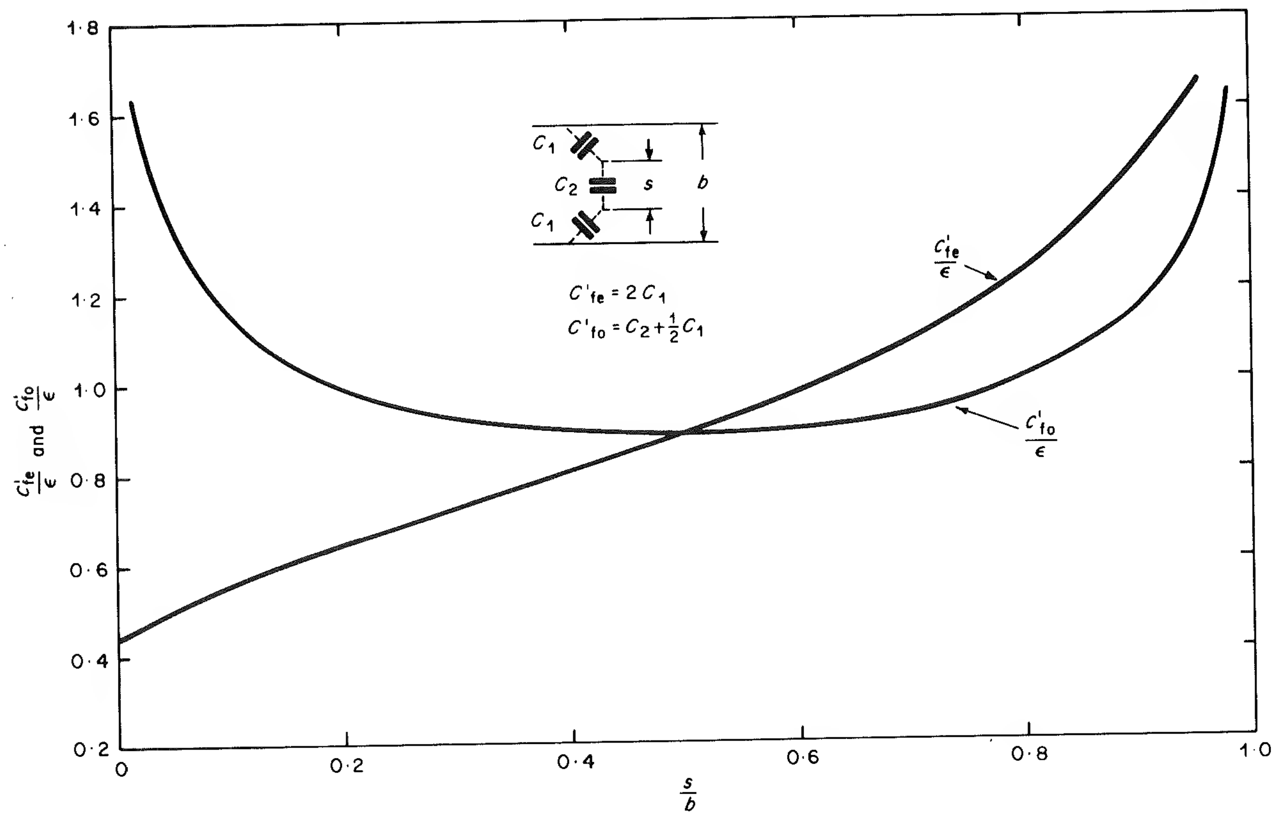


Fig. 3 - Fringing capacitances vs s/b

where ϵ_r = dielectric constant of medium
f_c = centre frequency of coupling bandwidth

To cover the Band IV, V region (470 – 854 MHz) l = 7·85 cm (3·1 in.). To prevent spurious modes (i.e. non TEM) propagating in the coupler we stipulate $b \ll \lambda_{min}$

and in general make $b \ll \frac{\lambda_{min}}{10}$

In this case $b < 2{\cdot}18$ cm.

Choose b = 1·02 cm. (0·4 in.)

For an Output Ratio of –6 dB, $K = \frac{1}{2}$, the coupling can be calculated from

$$K = \frac{c}{\sqrt{1-c^2}} \qquad (12)$$

giving c = 0·4467

and using Equations (5) and (6) gives

$$Z_{oe} = 80{\cdot}85\Omega \qquad Z_{oo} = 30{\cdot}92\Omega$$

By choosing a value for s, it is possible to substitute into Equations (9) and (10) to get two values for w/b.

Taking s = 0·254 cm (0·1 in.) gives from Z_{oe}; $\frac{w}{b}$ = 0·566

and from Z_{oo}; $\frac{w}{b}$ = 0·608

The difference is due to the approximations of the formulae. An arithmetic average gives w/b = 0·587. Thus for a 6 dB coupler the dimensions are:–

l = 7·85 cm; b = 1·02 cm; w = 0·6 cm (0·235 in);
s = 0·254 cm.

The width of the coupler, w', is chosen so as not to effect the odd/even impedances; a value w' = 2·54 cm (1·0 in.) is reasonable. Experimental tests showed that increasing w to 0·635 cm gave optimum results.

2.3. Design of 0 dB coupler

Using the same procedure as in 2·2 a 0 dB coupler was designed, the calculations giving the results:

l = 7·85 cm; b = 1·02 cm; w = 0·41 cm (0·16 in);
s = 0·08 cm (1/32 in.).

2.4. Construction

The method of construction is similar for both 0 dB and 6 dB couplers and is illustrated in Fig. 4. Full drawings are available under RA22481 (6 dB coupler) and RA22480 (0 dB coupler).

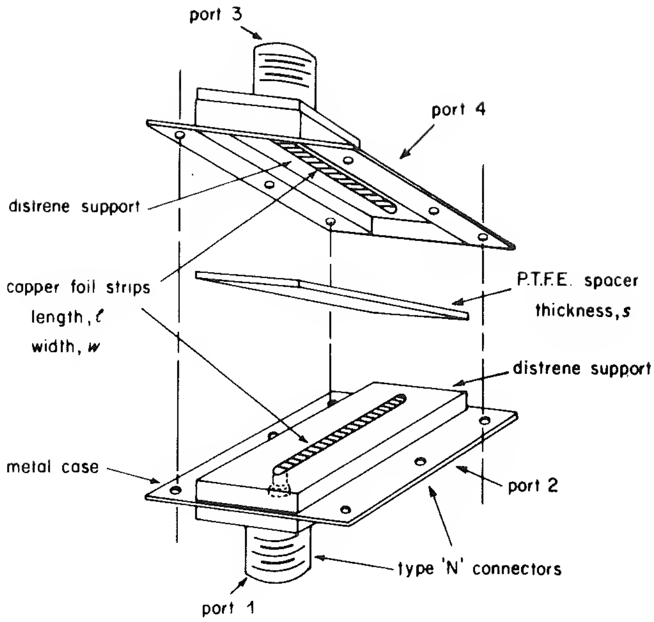


Fig. 4 - Construction details of 6 dB and 0 dB couplers

2.5. Performance

The graphs of Fig. 5 and Fig. 6 show typical performances of these couplers. Directivity of 30 dB, coupling to ±0·75 dB and reflection coefficient <4% are readily obtainable.

3. Coplanar strips

3.1. Formulae

Solutions for this configuration have been derived, again using conformal mapping techniques, by Cohn[3]. For zero-thickness strips the odd and even mode impedances are given by

$$Z_{oe} = \frac{30\pi}{\sqrt{\epsilon_r}} \frac{K(k'_e)}{K(k_e)} \qquad (13)$$

$$Z_{oo} = \frac{30\pi}{\sqrt{\epsilon_r}} \frac{K(k'_o)}{K(k_o)} \qquad (14)$$

where ϵ_r is the relative dielectric constant of the material filling the cross section, and

$$k_e = \tanh\left(\frac{\pi}{2}\frac{w}{b}\right) \tanh\left(\frac{\pi}{2}\frac{w+s}{b}\right) \qquad (15)$$

$$k_o = \tanh\left(\frac{\pi}{2}\frac{w}{b}\right) \coth\left(\frac{\pi}{2}\frac{w+s}{b}\right) \qquad (16)$$

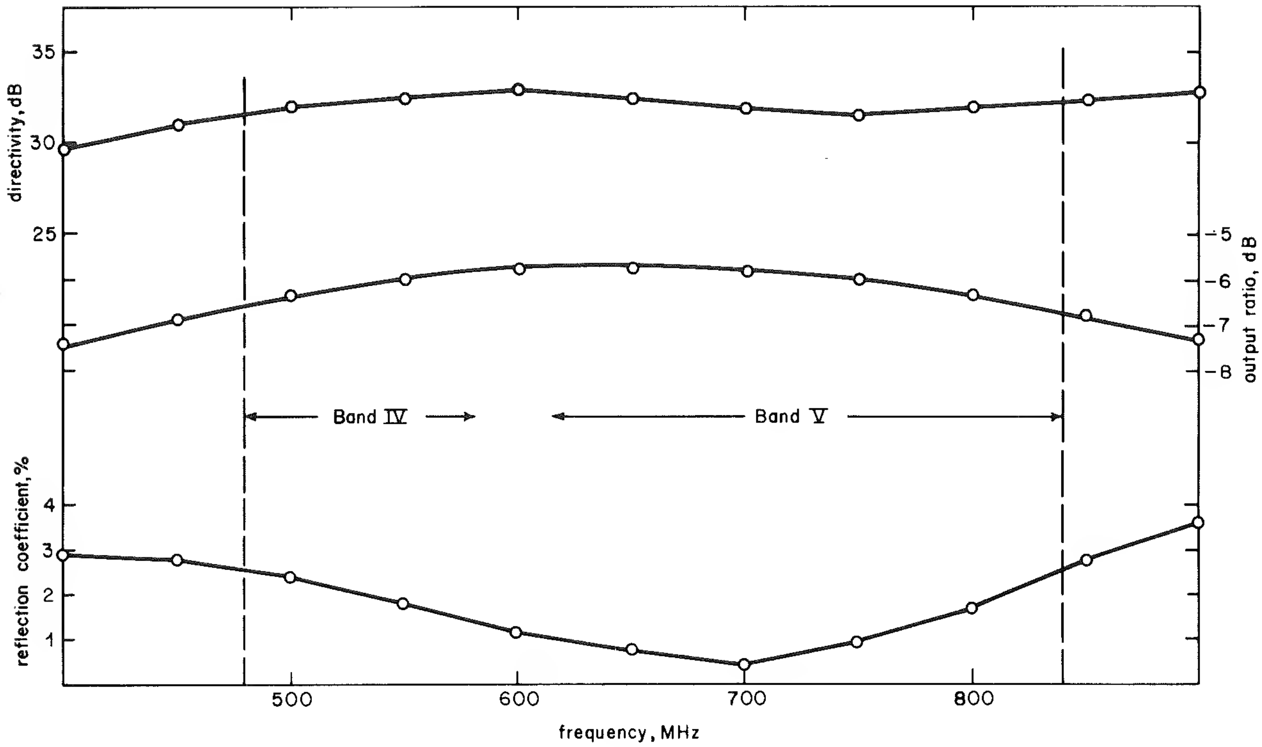


Fig. 5 - Performance of 6 dB coupler

$$k'_e = \sqrt{1 - k_e^2} \qquad k'_o = \sqrt{1 - k_o^2} \qquad (17)\ (18)$$

where w, s and b are defined in Fig. 2(b). The ratios w/b and s/b are easily calculated from:

$$\frac{w}{b} = \frac{2}{\pi} \tanh^{-1} \sqrt{k_e k_o} \qquad (19)$$

and

$$\frac{s}{b} = \frac{2}{\pi} \tanh^{-1} \left(\frac{1 - k_o}{1 - k_e} \sqrt{\frac{k_e}{k_o}} \right) \qquad (20)$$

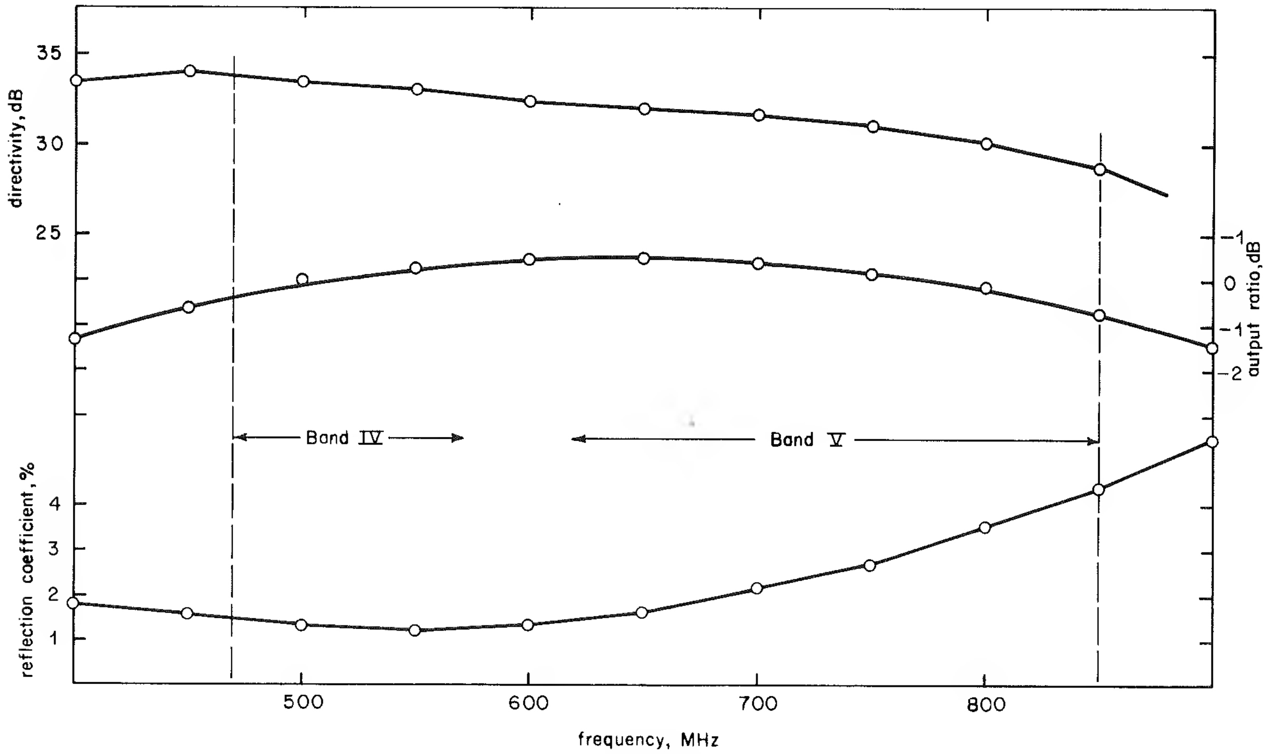


Fig. 6 - Performance of 0 dB coupler

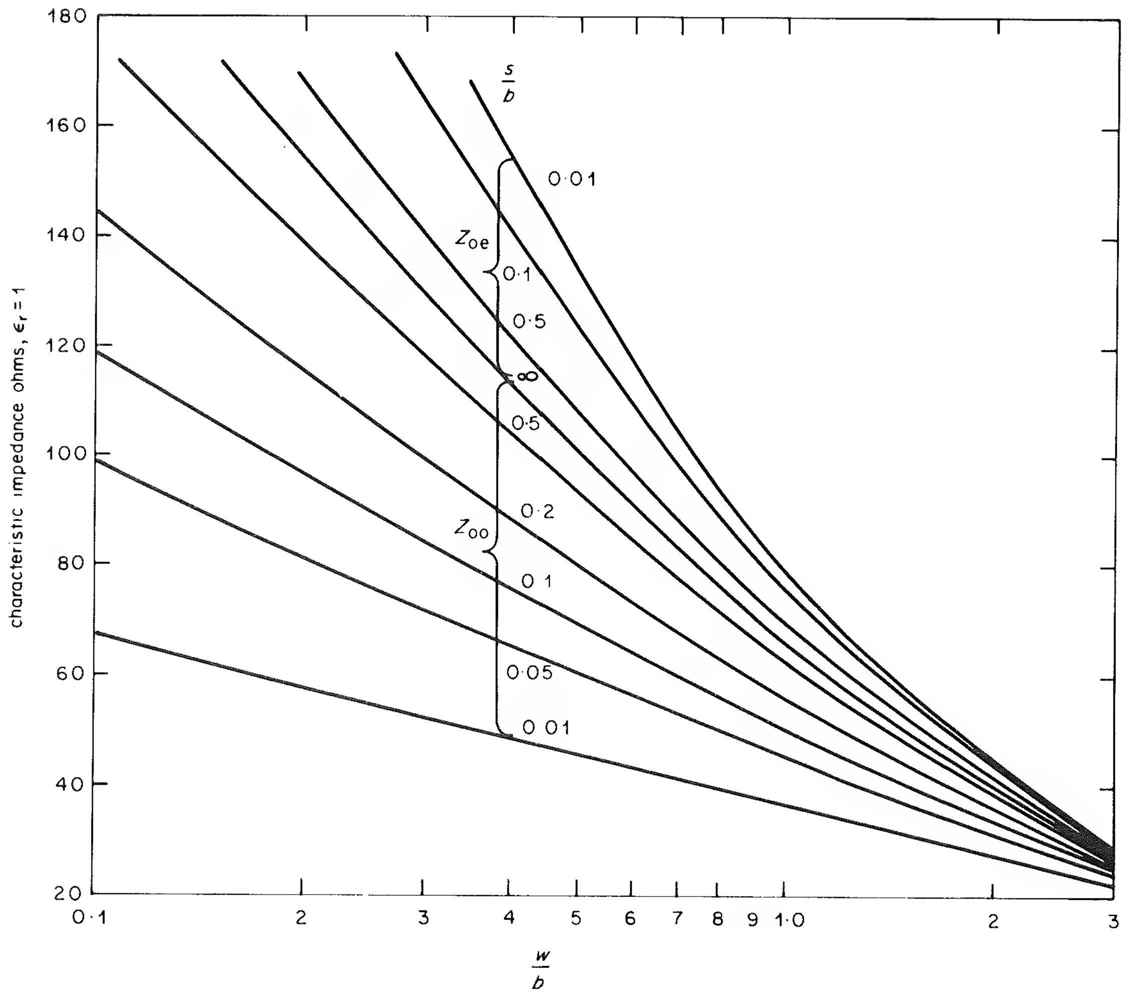


Fig. 7 - Graphs of Z_{oe}, Z_{oo} vs. w/b, s/b

k_e k_o can be obtained as functions of Z_{oe}, Z_{oo} from a graph of Equations (13) or (14). For the purposes of design work the graphs and nomograms in Reference 3 are sufficient and are reproduced in Figs. 7, 8 and 9.

3.2. Design of 15 dB coupler

The coplanar configuration of Fig. 2(*b*) is ideal for stripline construction. This is made full use of in the following design.

As in Section 2.2 the length of the coupling region is obtained from Equation (11) giving l = 5·94 cm, for MG51 (a silicon based, low loss material with an ϵ_r = 3·7). Applying the same constraint to prevent non TEM modes propagating as in the case of the 6 dB coupler, choose b = 0·635 cm (0·25 in.) or 4 layers of 1/16 in. P.C.B.

For an output ratio of −15 dB, K = 0·178 and using Equation (12) gives the midband coupling, c = 0·175.

The even and odd mode impedances are calculated from (5) and (6) and give

$$\sqrt{\epsilon_r} Z_{oe} = 114.8\Omega, \text{ and } \sqrt{\epsilon_r} Z_{oo} = 80.6\Omega$$

Using the nomograms of Figs. 8 and 9 values of w/b and s/b can be obtained — in this case,

$$w/b = 0{\cdot}52 \qquad s/b = 0{\cdot}215$$

and hence w = 0·33 cm and s = 0·14 cm.

Again, the width of the coupler is chosen so that it does not effect the even, odd impedances. A value of 6·0 cm is convenient.

3.3. Other considerations for stripline couplers

As will be seen in Fig. 10(*a*), which shows the construction of the stripline coupler, it is necessary to design short sections of 50Ω line to connect the coupler to the connectors. The dimensions of these sections must be carefully calculated and the interface of coaxial line to stripline must be optimised for minimum reflection coefficient.

3.3.1. Coaxial/stripline transition

The similarity between coaxial line and stripline suggested a connection as shown in Fig. 10(*b*) as this

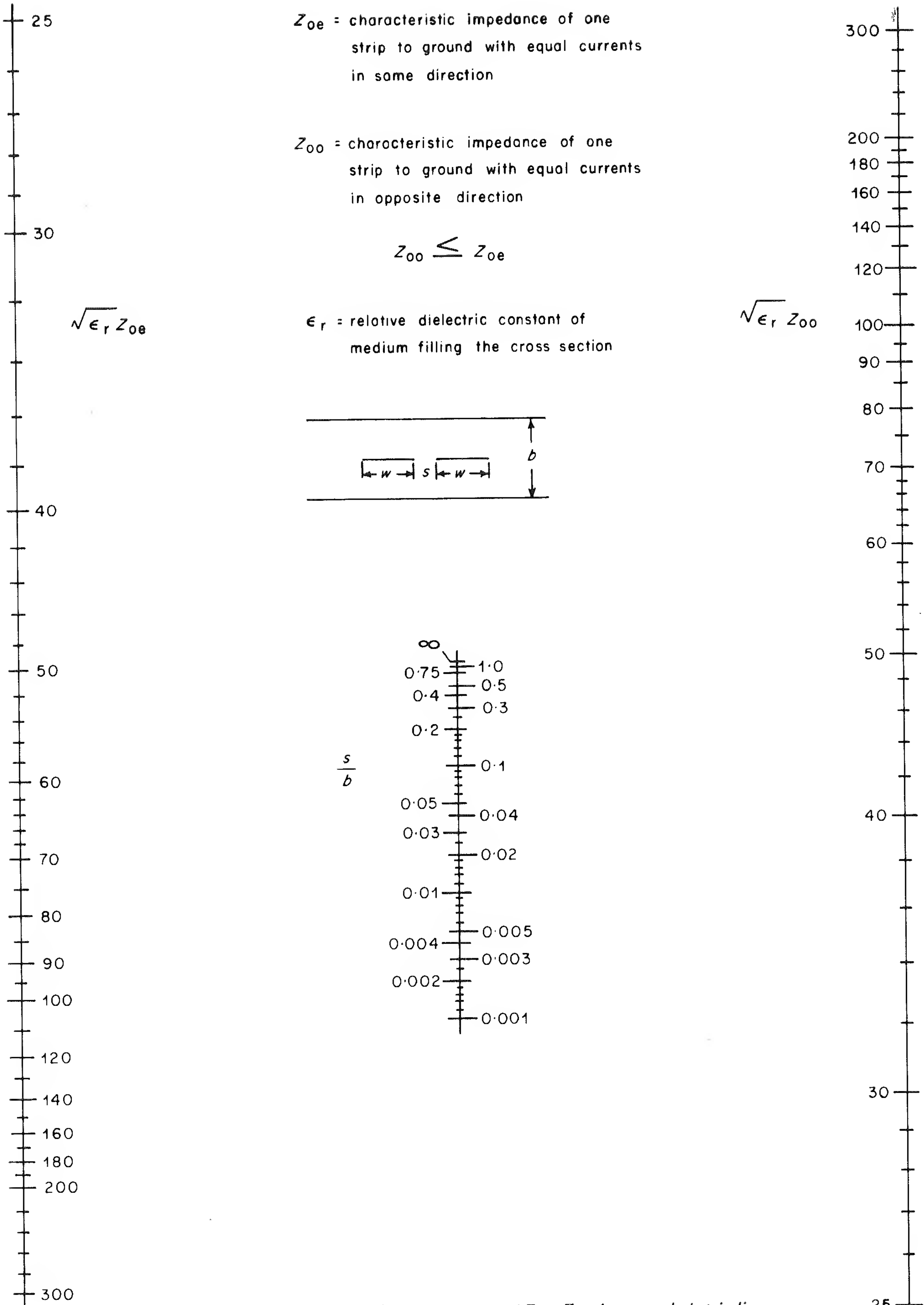


Fig. 8 - Nomogram giving s/b as a function of Z_{oe}, Z_{oo} in a coupled strip line

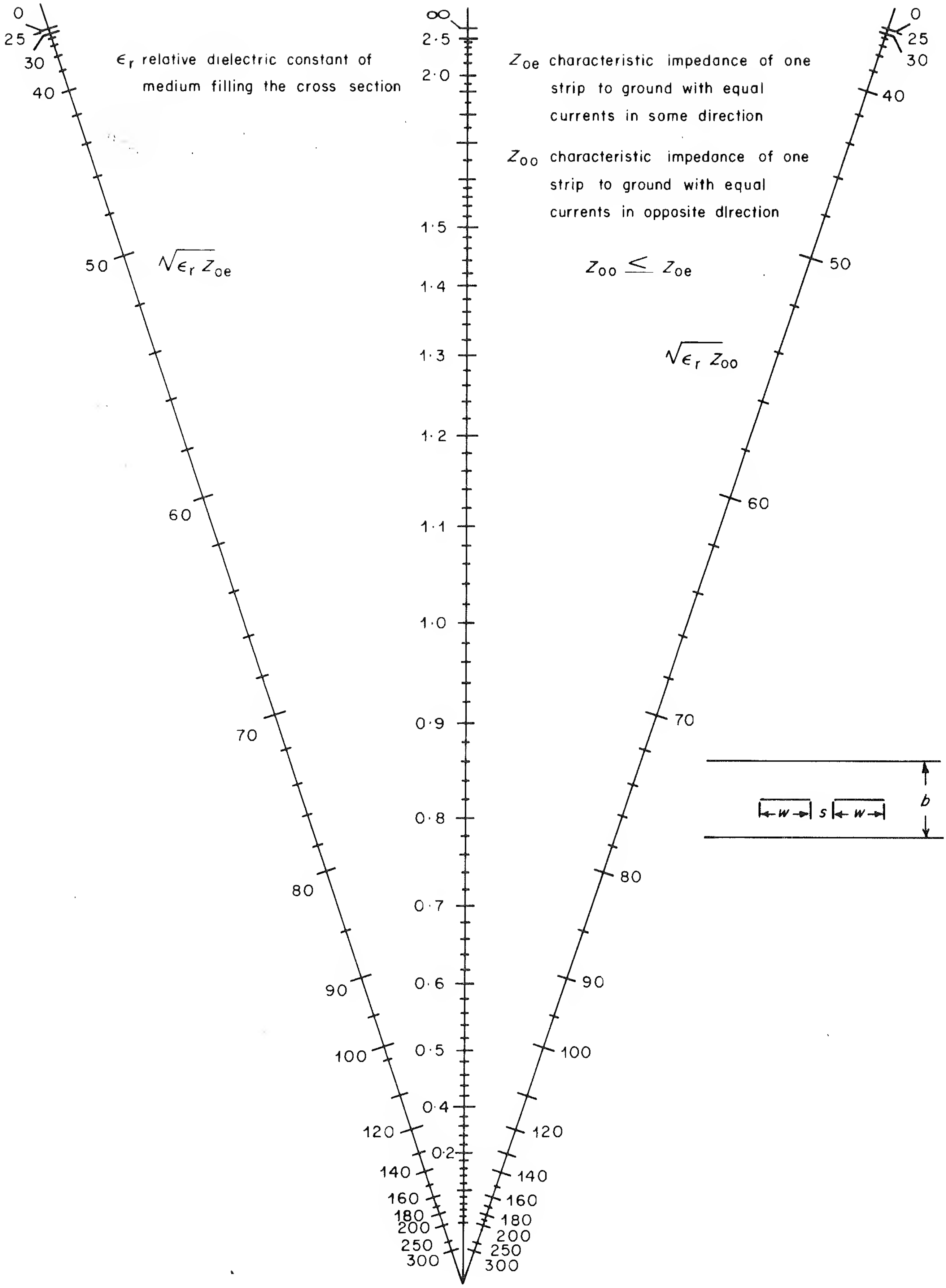
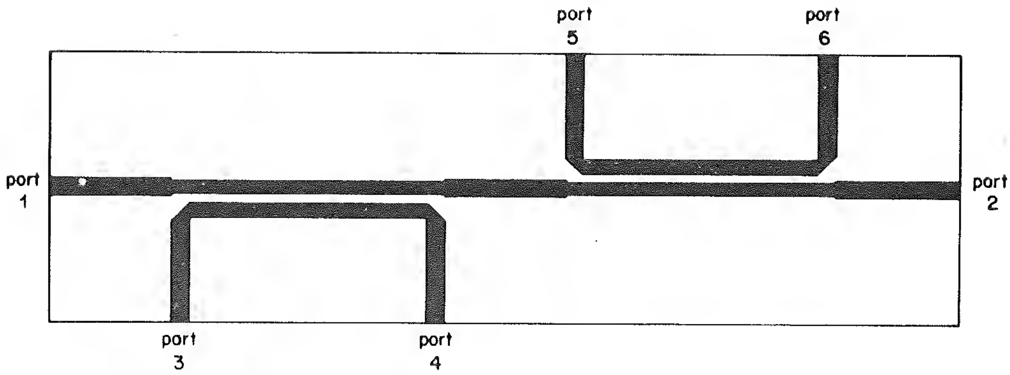
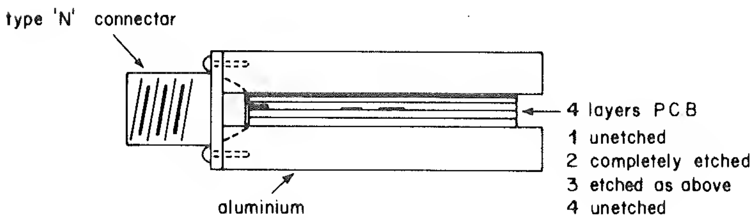


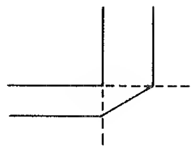
Fig. 9 - Nomogram giving w/b as a function of Z_{oe}, Z_{oo} in a coupled strip line



(a)



(b)



(c)

Fig. 10 - Construction of 15 dB coupler

requires the least re-orientation of wave field. To achieve a comparable match with other methods may require additional measures such as shorting pins between the ground planes.

3.3.2. Dimensions of stripline

Formulae for the dimensions of stripline for a given impedance have been developed by Barrett.[4] The parallel plate capacitance for three parallel conductors — C_p — as in Fig. 2(*b*) is

$$C_p = \frac{35{\cdot}4\left(\frac{w}{b}\right)\epsilon_r}{\left(1-\frac{\epsilon_r}{b}\right)} \text{ pF/m} \qquad (21)$$

For characteristic impedances up to 100Ω the fringing capacitance, C_f must be taken into account

$$\therefore\ C_{tot} = C_p + C_f\epsilon_r \qquad (22)$$

It has been found experimentally C_f is constant at 15 pF/m for $w/b > 0{\cdot}3$ and since

$$Z_o = \frac{\sqrt{\mu\epsilon_r}}{3.10^8\, C_{tot}} \qquad (23)$$

then $$Z_o = \frac{\sqrt{\mu\epsilon_r}\left(1-\frac{t}{b}\right)}{3.10^8\left[35{\cdot}4\frac{w}{b} + \left(1-\frac{t}{b}\right)C_f\right]\epsilon_r}$$

which for zero thickness strips is

$$Z_o = \frac{\sqrt{\mu\epsilon_r}}{3.10^8\left[35{\cdot}4\,\frac{w}{b} + C_f\right]\epsilon_r\, 10^{-12}} \qquad (24)$$

For a characteristic impedance of 50Ω this gives w/b = 0·540 for MG51 giving w = 0·34 cm.

3.3.3. Design of bends

The right angle bends in the stripline are mitred to give minimum reflections.[5] This is illustrated in Fig. 10(*c*).

3.4. Construction

The prototype coupler was in fact a dual 15 dB coupler designed around printed circuit techniques. Fig. 10(*a*) shows the arrangements of the printed couplers and the 'sandwich' method of construction is shown in Fig. 10(*b*). Full drawings are available under R22499 A4.

3.5. Performance

Fig. 11 shows that directivities of approximately 25 dB, reflection coefficients less than 4% and coupling to within 1 dB were obtained.

4. Conclusions

The methods of design for two basic configurations cover a wide range of coupling and use economical construction methods. The performance criterion of reflection

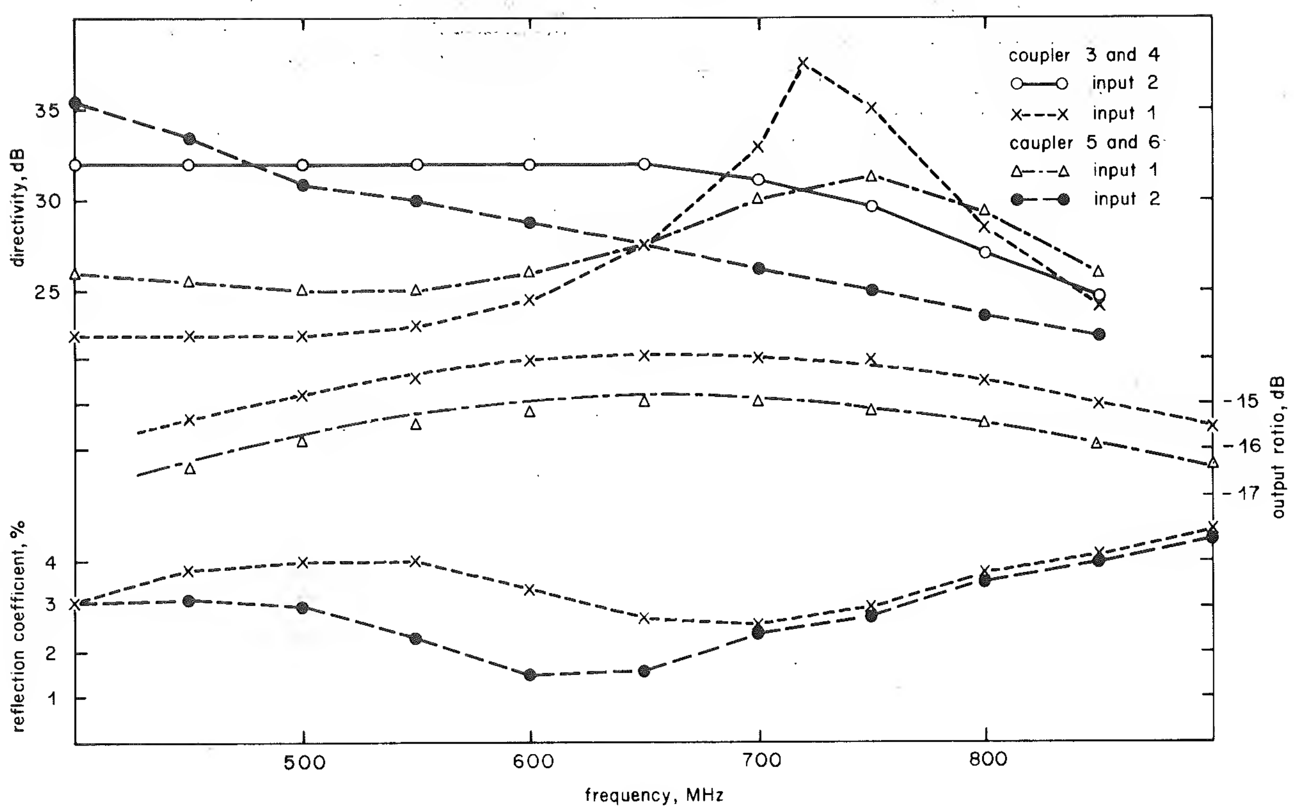


Fig. 11 - Performance of 15 dB coupler

coefficient less than 3%, directivity greater than 25 dB and coupling ±1 dB were satisfied.

5. References

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6. Acknowledgements

The author wishes to thank the McGraw-Gill Book Company for permission to reprint Fig. 12 taken from Reference 7, copyright 1964 McGraw Hill Inc., and also the IEEE for permission to reprint Figs. 3, 7, 8, 9 taken from Reference 1, 3.

Appendix I

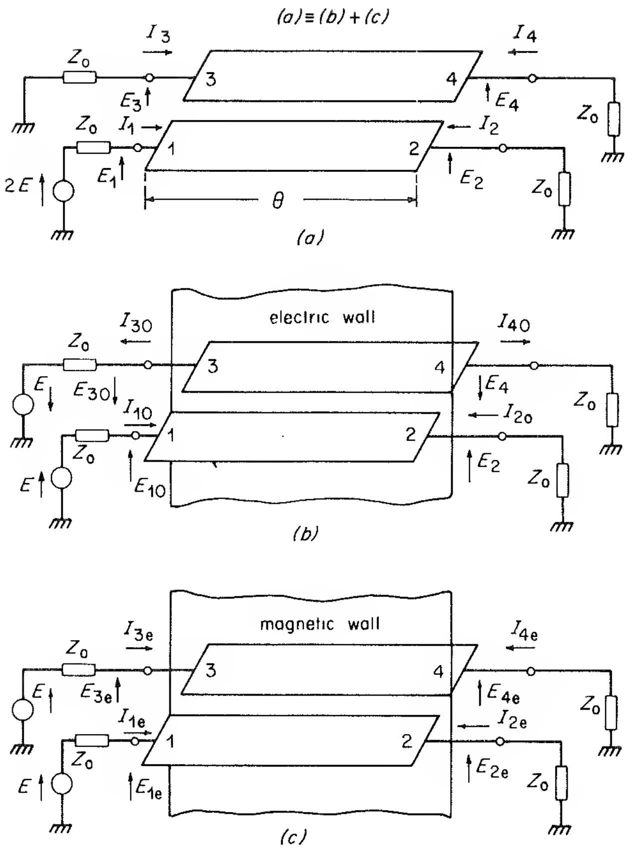


Fig. 12 - Even and odd excitation of a directional coupler

Fig. 12(a) shows a directional coupler correctly terminated and excited by a voltage $2E$. By superposition it can be seen that this is equivalent to the behaviour of the even and odd modes. In order for this coupler to be matched at all impedances the input impedance, $Z_{in} = E_1/I_1$ must equal Z_o.

From Fig. 12:

$$Z_{in} = \frac{E_{1o} + E_{1e}}{I_{1o} + I_{1e}} = \frac{\dfrac{Z_{1o}\,E}{Z_o + Z_{1o}} + \dfrac{Z_{1e}\,E}{Z_o + Z_{1e}}}{\dfrac{E}{Z_o + Z_{1o}} + \dfrac{E}{Z_o + Z_{1e}}} \tag{25}$$

Z_{1o}, Z_{1e} are the impedances of a length of transmission line terminated in Z_o, i.e.

$$Z_{1o} = Z_{oo}\frac{Z_o + jZ_{oo}\tan\theta}{Z_{oo} + jZ_o\tan\theta} \tag{26}$$

$$Z_{1e} = Z_{oe}\frac{Z_o + jZ_{oe}\tan\theta}{Z_{oe} + jZ_o\tan\theta} \tag{27}$$

If $Z_{in} = Z_o$

i.e. $$\frac{Z_{1o}(Z_o + Z_{1e}) + Z_{1e}(Z_o + Z_{1o})}{Z_o + Z_{1e} + Z_o + Z_{1o}} = Z_o$$

$$Z_oZ_{1o} + 2Z_{1o}Z_{1e} + Z_{1e}Z_o = 2Z_o^2 + Z_{1e}Z_o + Z_{1o}Z_o$$

$$Z_o^2 = Z_{1o}Z_{1e}$$

Using (26) and (27) $Z_o^2 = Z_{oo}Z_{oe}$ for all θ

$$Z_o = \sqrt{Z_{oo}Z_{oe}} \tag{28}$$

Under this condition, the voltage at terminal 1 is E. The voltages $E_{3e} = E_{1e}$ and $E_{2e} = E_{4e}$ can be determined from the analysis of a transmission line of length θ, characteristic impedance Z_{oe}, terminated at either end by an impedance $(Z_{oo}Z_{oe})^{1/2}$ and fed by a voltage E. This can be repeated for $E_{2o} = -E_{4o}$ and $E_{3o} = -E_{1o}$ and using

$$E_3 = E_{3e} - E_{3o}$$

$$\frac{E_3}{E} = \frac{j\left(\sqrt{\dfrac{Z_{oe}}{Z_{oo}}} - \sqrt{\dfrac{Z_{oo}}{Z_{oe}}}\right)\sin\theta}{2\cos\theta + j\left(\sqrt{\dfrac{Z_{oe}}{Z_{oo}}} + \sqrt{\dfrac{Z_{oo}}{Z_{oe}}}\right)\sin\theta}$$

$$E_4 = E_{4e} - E_{4o} = 0 \tag{29}$$

$$E_2 = E_{2e} - E_{2o}$$

$$\frac{E_2}{E} = \frac{2}{2\cos\theta + j\left(\sqrt{\dfrac{Z_{oe}}{Z_{oo}}} - \sqrt{\dfrac{Z_{oo}}{Z_{oe}}}\right)\sin\theta}$$

$$\text{If } c = \frac{Z_{oe}/Z_{oo} - 1}{Z_{oe}/Z_{oo} + 1} \tag{30}$$

$$\frac{E_3}{E} = \frac{j\,c\sin\theta}{\sqrt{1 - c^2}\cos\theta + j\sin\theta}$$

$$\frac{E_2}{E} = \frac{\sqrt{1 - c^2}}{\sqrt{1 - c^2}\cos\theta + j\sin\theta}$$

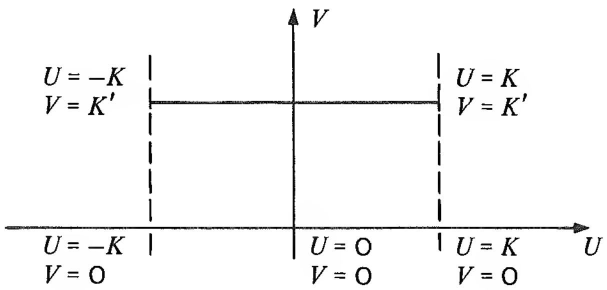
(28) and (30) give the result

$$Z_{oe} = Z_o\sqrt{\frac{1+c}{1-c}} \qquad Z_{oo} = Z_o\sqrt{\frac{1-c}{1+c}}$$

Appendix II

The solution for the impedances of three-conductor systems is achieved by conformal mapping. The essence is to 'map' the unknown field into a known configuration. As all partial capacitances are equal to those between their images in the map, detailed study of field distribution becomes unnecessary.

A convenient known field is that of Fig. 13 for which



$$Z_o = \frac{\mu\epsilon}{C}$$

$$= 120\pi\frac{K'}{2K}\ \text{ohms.}$$

Fig. 13

The properties of conformal mapping means that the characteristic impedance of a given cross section is not changed by successive transformations. After mapping the unknown field of the conductor system into the known field, the impedance is unaltered and it remains to relate K, K' to the dimensions of the coupled line.

To do this, a 'mapping derivative' is obtained by the Schwarz Christoffel method, which transforms from the Z plane to the w plane viz:—

$$\frac{dz}{d\omega} = N\Pi\alpha(w - U\alpha)^{-\gamma\alpha} \qquad 2 = N\int \Pi\alpha(w - U\alpha)^{-\gamma\alpha}dw + M$$

When this expression is evaluated, integration gives relations relating points on one plane with their images in the other. Thus the characteristic impedance can be expressed in terms of the physical parameters of the cross-section.

With a three conductor system however it is common for the mapping derivative to have up to four poles and hence most of the integrals are elliptic. The treatment of these integrals is extremely mathematical and beyond the scope of this report.

Details of derivations can be obtained from the references.

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